

# DP Mathematics

## *Curriculum Review*

Summary report to teachers

April 2017

## Mathematics curriculum review: update for teachers

The mathematics curriculum review is currently in year 5 of the 7-year cycle. All Diploma Programme curriculums are reviewed on a seven-year cycle to ensure they are fit for purpose and incorporate the latest educational research, as well as lessons learned through evaluation of the existing curriculum. We aim to create robust curriculums which will remain relevant up to the year 2027. The current review is developing two new subjects which will replace the existing 4 subjects. The new subject guides for Mathematics: Analysis and approaches, and Mathematics: Applications and interpretation, will be published on the new Programme resource centre (PRC) which will replace the OCC and have their own websites in early 2019. These courses will have first teaching from August 2019 and first examinations in May 2021.

This document is intended to give teachers an overview of the development of the new subjects to date, to help with long term planning and resource investment ahead of the launch of the new subject guides and extensive teacher support materials in 2019.

## Mathematics curriculum review timeline

Year	Activity
2012/14	Two years of research and evaluation. Invitations issued to participants in the external review process. Research and evaluation report compiled by the curriculum manager. Questionnaire sent to schools in mid-2014
2014/15	Circulation of curriculum manager's report to review participants. First external review meeting November 2014. Internal review committee meetings in October and November 2014
2015/16	SL development meeting March 2015, HL development meeting April 2015. IA meeting September 2015 and February 2016. Joint development meetings October 2015 and March 2016
2016/17	Second survey to schools September 2016. Joint development meetings November 2016 and March 2017
2017/18	Development meeting and specimen paper development meeting planned for October/November 2017
2019	New guides, teacher support materials, videos and specimen papers to be published on the Programme resource centre (which will replace the OCC) in January 2019. Subject specific seminars (SSS) will take place to launch the new courses. First teaching of the new subjects
2021	First examination of the new subjects (November 2020 last examinations of the old subjects)

## Summary of the major discussions relating to the subjects:

### Subject descriptors and subject names

**Mathematics: Analysis and approaches** at SL and HL is appropriate for students who enjoy developing their mathematics to become fluent in the construction of mathematical arguments and develop strong skills in mathematical thinking. They will also be fascinated by exploring real and abstract applications of these ideas, with and without the use of technology. Students who take **Mathematics: Analysis and approaches** will be those who enjoy the thrill of mathematical problem solving and generalization. This subject is aimed at students who will go on to study subjects with substantial mathematics content such as mathematics itself, engineering, physical sciences, or economics for example.

**Mathematics: Applications and interpretation** SL and HL is appropriate for students who are interested in developing their mathematics for describing our world and solving practical problems. They will also be interested in harnessing the power of technology alongside exploring mathematical models. Students who take **Mathematics: Applications and interpretation** will be those who enjoy mathematics best when seen in a practical context. This subject is aimed at students who will go on to study subjects such as social sciences, natural sciences, statistics, business, some economics, psychology, and design, for example.

The subject names have been chosen to reflect the content and aims of the subjects, withstand translation between the 3 working languages of the IB, English, French and Spanish, and reflect the different emphasis of each subject. **Mathematics: Analysis and approaches** reflects the emphasis on calculus and on algebraic, graphical and numerical approaches. **Mathematics: Applications and interpretation** emphasises the applied nature of the subject, and also that interpretation of results in context is an important element of the subject.

### SL as a subset of HL

To bring DP mathematics in line with other subjects within the diploma the SL content will become a complete subset of the HL content in both subjects. All students will complete 120 hours of SL content and 30 hours of investigation, inquiry and problem-solving activities including completing the internal assessment. HL students will complete a further 90 hours of additional higher level content. SL as a subset of HL is being designed to allow more flexibility in the way that schools group their students and to encourage a greater proportion of students to take a HL mathematics course.

## Common content in both subjects

To ensure alignment between the two subjects there will be 60 hours of common SL content. This will allow schools to be more flexible in their timetabling. It may also assist teachers who teach SL and HL students together in one group or schools where both subjects may be taught in the same classroom.

## Thirty hours for inquiry, problem-solving and the internal assessment task

Within both subjects at SL and HL there will be a recommended 30 hours in which students can engage in extended work on areas of mathematics not easily assessed in examinations. This will allow students to develop the skills necessary to grow as mathematicians and to approach the internal assessment with the required skill set.

## Aims

The aims of both new subjects remain similar to those of the current subjects, however with some small changes to bring them up to date and create better alignment with MYP Mathematics. Two new aims (11 and 12) have been added to ensure that development of the IB Learner Profile attributes and Approaches to Teaching and Learning are more explicit within the subjects. The aims are now:

The mathematics courses aim to contribute to students' personal attributes, subject understanding and global awareness by enabling them to:

1. develop a curiosity and enjoyment of mathematics, and appreciate its elegance and power
2. develop an understanding of the concepts, principles and nature of mathematics
3. communicate mathematics clearly, concisely and confidently in a variety of contexts
4. develop logical and creative thinking, and patience and persistence in problem solving to instill confidence in using mathematics
5. employ and refine their powers of abstraction and generalization
6. take action to apply and transfer skills to alternative situations, to other areas of knowledge and to future developments in their local and global communities
7. appreciate how developments in technology and mathematics influence each other
8. appreciate the moral, social and ethical questions arising from the work of mathematicians and its applications
9. appreciate the universality of mathematics and its multicultural, international and historical perspectives
10. appreciate the contribution of mathematics to other disciplines, and as a particular "area of knowledge" in the TOK course

11. develop the ability to reflect critically upon their own work and the work of others
12. independently and collaboratively extend their understanding of mathematics

### Assessment objectives

The assessment objectives for both subjects will remain largely unchanged from the current assessment objectives with some small changes.

Objective 2 will be "**Problem solving**: recall, select and use their knowledge of mathematical skills, results and models in both abstract and *real world* contexts to solve problems". This will highlight the authentic nature of the contexts.

Objective 3, "**Communication and interpretation**", will include the addition of the phrase "use appropriate notation and terminology", to accommodate changes to mathematical notation due to the increasing use of technology.

Objective 6 was previously "inquiry approaches" for Mathematics HL and SL, and "investigative approaches" for Mathematical studies SL. This will now become "**Inquiry Approaches**: investigate unfamiliar situations, both abstract and from the real-world, involving organizing and analyzing primary and secondary data, making conjectures, drawing conclusions, and considering their validity".

### External assessment

**Mathematics: Analysis and approaches SL** will be assessed with two written papers. Each paper will consist of a section A (short questions) and a section B (long questions). Paper 1 will be without the use technology and paper 2 will allow the use of a graphical calculator.

**Mathematics: Applications and interpretation SL** will be assessed with two written papers both of which will require the use of the technology in the form of a graphical calculator. Paper 1 will consist of short questions and paper 2 will consist of longer questions.

**Both HL courses** will be assessed in a similar way to their corresponding SL courses on papers 1 and 2, and in addition will have a paper 3.

The HL paper 3 in both subjects may be a one hour problem solving paper possibly requiring the use of spreadsheets and graphing software. There may be two extended questions, based on content from the syllabus but leading to generalization or interpretation of the problems. A questionnaire will be sent to schools in August 2017 to assess the extent of schools' access to this type of technology.

	<b>SL</b>	<b>% weighting</b>	<b>HL</b>	<b>% weighting</b>
Paper 1	1.5 hrs	40	2 hrs	30
Paper 2	1.5 hrs	40	2 hrs	30
Paper 3	n/a	n/a	1 hr	20
Internal assessment: Investigative, problem solving and modelling skills development leading to one written exploration	30 hrs	20	30 hrs	20
Total		100		100

It is hoped to trial papers 1, 2 and 3 in August 2017 with schools in the IB Asia Pacific region and we are currently seeking participants for this trial. Due to the nature of the papers we are seeking November session schools with candidates taking HL Mathematics with the calculus option. If you are interested in participating in this trial please contact: [dpgroup4and5@ibo.org](mailto:dpgroup4and5@ibo.org).

### Internal assessment (IA)

The internal assessment task will be the same for both HL and SL and will be based on the current Mathematics SL and Mathematics HL exploration model. This model has been shown to be a robust and reliable assessment tool and enables students to undertake a piece of research which interests them and models the type of mathematical activity undertaken in the modern world.

Both subjects will have the same assessment criteria based on the current Mathematics SL and HL criteria. Both SL and HL will have the same assessment criteria A to D with criteria E "Use of Mathematics" being slightly different for HL and SL reflecting the different demands.

We anticipate that regardless of which subject students are following they may produce an exploration suitable to the other subject. A trial was carried out in August 2016 to test the assessment criteria to ensure fairness and comparability between the types of explorations that students are likely to produce and the consistency of marking between levels and markers. The trial indicated that the new criteria were reliable and could be

applied more consistently than the current criteria. It also indicated that students who produce a Mathematical studies SL type “project” would not be penalized using the proposed criteria.

## Prior learning

Many schools have already begun to think about the mathematics that their students will need to be well prepared for the IB DP Mathematics courses in 2019. Below is a provisional but non-exhaustive list which gives an indication of the mathematics a student should have covered prior to beginning the new DP mathematics courses. It is not expected that students will begin the course with in-depth knowledge of all the prior learning but that they are at least familiar with the vocabulary and basic concepts.

Students completing HL mathematics courses may have mastered these concepts to a greater level than SL students.

- Use of addition, subtraction, multiplication and division using integers, decimals and fractions, including order of operations
- Prime numbers, factors (divisors) and multiples
- Ratio, percentage and proportion.
- Number systems: natural numbers; integers,  $\mathbb{Z}$ ; rationals,  $\mathbb{Q}$ , and irrationals; real numbers,  $\mathbb{R}$
- SI (*Système International*) units of mass, time, length, area and volume
- Rounding, decimal approximations and significant figures
- Definition and elementary treatment of absolute value (modulus)
- Manipulation of algebraic expressions, including factorization and expansion
- Rearranging formulae
- Evaluating expressions by substitution
- Evaluating exponential expressions
- Use of inequalities
- Solving linear equations and inequalities
- Simplification of expressions
- Expression of numbers in the form,  $a \times 10^k$ ,  $1 \leq a < 10$ ,  $k \in \mathbb{Z}$
- Familiarity with commonly accepted world currencies
- Solving systems of equations in two variables
- Graphing linear and quadratic functions using technology
- Mappings of the elements of one set to another. Illustration by means of ordered pairs, tables, diagrams and graphs
- The Cartesian plane: ordered pairs  $(x, y)$ , origin, axes

- Elementary geometry of the plane including the concepts of dimension for point, line, plane and space. The equation of a line in the form  $y=mx +c$
- Informal consideration of parallel and perpendicular lines, including  $m_1 = m_2$ , and  $m_1m_2 = -1$
- Pythagoras' theorem and its converse
- Mid-point of a line segment and distance between two points in the Cartesian plane
- Geometry of simple plane figures
- Angle measurement in degrees
- Right-angle trigonometry, including simple applications for solving triangles
- Simple geometric transformations: translation, reflection, rotation, enlargement, including the concept of scale factor of an enlargement
- The circle, its centre and radius, area and circumference
- Perimeter and area of plane figures. Properties of triangles and quadrilaterals, including parallelograms, rhombuses, rectangles, squares, kites and trapeziums (trapezoids); compound shapes
- Familiarity with three-dimensional shapes (prisms, pyramids, spheres, cylinders and cones)
- Volumes and surface areas of cuboid, right prism, right pyramid, right cone, cylinder, sphere, hemisphere and compound 3d shapes.
- Compass directions
- The collection of data and its representation in bar charts, pie charts, pictograms, and line graphs
- Obtaining simple statistics from discrete data, including mean, median, mode, range
- Calculating simple probabilities.

## The guide

The new guides will be available as separate websites for both subjects and as downloadable PDFs. The websites will include extensive teacher support materials including specimen papers, videos and IA exemplars, and may also include, for example, unit plans, exemplar questions with worked solutions, suggested activities, and guidance on modelling and inquiry.

The layout of the content sections in the current guide with three columns was considered to be appropriate and the most informative for teachers, students, examiners and university recognition. Each topic will begin with a list of key concepts, the overarching and the topical conceptual understandings. This is to support the conceptual understanding of each topic and to encourage teachers to make explicit the importance and relevance of the mathematics.



## Allocation of hours – approximate at time of writing

	Mathematics: Analysis and approaches		Mathematics: Applications and interpretation	
	SL	HL	SL	HL
Number and algebra	18	34	18	34
Functions	21	31	26	36
Trigonometry and geometry	26	54	16	46
Statistics and probability	27	36	40	52
Calculus	28	55	20	42
“Toolkit” and IA	30	30	30	30
Total	150	240	150	240

### The current options

It has been decided not to include options in the new HL courses due to the very low uptake of two of the four options. Currently the most popular options are calculus, and statistics and probability. Both new subjects will include aspects of these current options as part of the additional HL content, with Mathematics: Analysis and approaches containing more of the calculus option and Mathematics: Applications and interpretation containing more of the statistics and probability option, as well as some of the discrete option.

### Further mathematics HL

Further mathematics HL will be discontinued and will have last examination in May 2020. This decision was taken due to the very small number of students taking the subject and the very small number of schools offering the subject. It is intended that by offering two HL mathematics courses this will provide a coherent structure and a greater choice to support a larger number of able mathematicians within the Diploma Programme. It was felt that this aim could not be achieved while continuing to offer Further mathematics.

## The Content

The content is still under development and items below may be subject to change when the guides are published:

### Mathematics: Analysis and approaches

The **number and algebra SL** looks at: scientific notation, arithmetic and geometric sequences and series and their applications including financial applications, laws of logarithms and exponentials, solving exponential equations, simple proof, approximations and errors, and the binomial theorem. The **number and algebra HL** looks at: permutations and combinations, partial fractions, complex numbers, proof by induction, contradiction and counter-example, and solution of systems of linear equations.

The **functions SL** looks at: equations of straight lines, concepts and properties of functions and their graphs, including composite, inverse, the identity, rational, exponential, logarithmic and quadratic functions. Solving equations both analytically and graphically, and transformation of graphs. The **functions HL** looks at: the factor and remainder theorems, sums and products of roots of polynomials, rational functions, odd and even functions, self-inverse functions, solving function inequalities and the modulus function.

The **geometry and trigonometry SL** looks at: volume and surface area of 3d solids, right-angled and non-right-angled trigonometry including bearings and angles of elevation and depression, radian measure, the unit circle and Pythagorean identity, double angle identities for sine and cosine, composite trigonometric functions, solving trigonometric equations. The **geometry and trigonometry HL** looks at: reciprocal trigonometric ratios, inverse trigonometric functions, compound angle identities, double angle identity for tangent, symmetry properties of trigonometric graphs, vector theory, applications with lines and planes, and vector algebra.

The **statistics and probability SL** looks at: collecting data and using sampling techniques, presenting data in graphical form, measures of central tendency and spread, correlation, regression, calculating probabilities, probability diagrams, the normal distribution with standardization of variables, and the binomial distribution. The **statistics and probability HL** looks at: Bayes theorem, probability distributions, probability density functions, expectation algebra.

The **calculus SL** looks at: informal ideas of limits and convergence, differentiation including analysing graphical behaviour of functions, finding equations of normals and tangents, optimisation, kinematics involving displacement, velocity, acceleration and total distance travelled, the chain, product and quotient rules, definite and indefinite integration. The **calculus HL** looks at: introduction to continuity and differentiability, convergence and divergence, differentiation from first principles, limits and L'Hopital's rule, implicit differentiation, derivatives of inverse and reciprocal trigonometric functions, integration by substitution and parts, volumes of revolution, solution of first order differential equations using Euler's method, by separating variables and using the integrating factor, Maclaurin series.

### Mathematics: Applications and interpretation

The **number and algebra SL** looks at: scientific notation, arithmetic and geometric sequences and series and their applications in finance including loan repayments, simple treatment of logarithms and exponentials, simple proof, approximations and errors. The **number and algebra HL** looks at: laws of logarithms, complex numbers and their practical applications, matrices and their applications for solving systems of equations, for geometric transformations, and their applications to probability.

The **functions SL** looks at: creating, fitting and using models with linear, exponential, natural logarithm, cubic and simple trigonometric functions. The **functions HL** looks at: use of log-log graphs, graph transformations, creating, fitting and using models with further trigonometric, logarithmic, rational, logistic and piecewise functions.

The **geometry and trigonometry SL** looks at: volume and surface area of 3d solids, right-angled and non-right-angled trigonometry including bearings, surface area and volume of composite 3d solids, establishing optimum positions and paths using Voronoi diagrams. The **geometry and trigonometry HL** looks at: vector concepts and their applications in kinematics, applications of adjacency matrices, and tree and cycle algorithms.

The **statistics and probability SL** looks at: collecting data and using sampling techniques, presenting data in graphical form, measures of central tendency and spread, correlation using Pearson's product-moment and Spearman's rank correlation coefficients, regression, calculating probabilities, probability diagrams, the normal distribution, Chi-squared test for independence and goodness of fit. The **statistics and probability HL** looks at: the

binomial and Poisson distributions, designing data collection methods, tests for reliability and validity, hypothesis testing and confidence intervals.

The **calculus SL** looks at: differentiation including analysing graphical behavior of functions and optimisation, using simple integration and the trapezium/trapezoidal rule to calculate areas of irregular shapes. The **calculus HL** looks at: kinematics and practical problems involving rates of change, volumes of revolution, setting up and solving models involving differential equations using numerical and analytic methods, slope fields, coupled and second-order differential equations in context.

### Note on examinations in the Diploma Programme

Developments in onscreen assessment throughout the IB Diploma Programme are ongoing. There is no timeline for mathematics to be examined on-screen yet, and any commitment to do so will be based on successful development and trialling of appropriate onscreen examination tools and functionality.

**All the above should be considered as a work in progress and may or may not reflect the material which will finally appear in the guides.**

Finally, a very big thank you to all the DP mathematics community who have been involved in roles however big or small during the review so far.

If you have any comments please email: [ibid@ibo.org](mailto:ibid@ibo.org)